Beam-Forming Electrostrictive Matrix

Eugene Danicki and Yuriy Tasinkevych

Abstract In this paper a two-dimensional ultrasound transducer comprising crossed periodic metal electrodes placed on both sides of electrostrictive layer and representing the matrix rows and columns is described. Such a system is capable of electronic beam-steering of generated wave both in elevation and azimuth. The wave-beam control is achieved by addressable driving of two-dimensional matrix transducer through proper voltage supply of electrodes on opposite surfaces of the layer. In this paper a semi-analytical method of analysis of the considered transducer is proposed, which is a generalization of the well-known BIS-expansion method. It was earlier exploited with great success in the theory of interdigital transducers of surface acoustic waves, theory of elastic wave scattering by cracks and certain advanced electrostatic problems. The paper presents evaluation of stress in the electrostrictive layer excited by potentials applied to electrodes. The corresponding nontrivial electrostatic problem is formulated. Some numerical examples showing the resulting generated electrostrictive stress in the dielectric layer are presented for a simple case of one upper strip excited by a uniform voltage and all bottom strips grounded.

Keywords Beamforming • Transducer array • BIS-expansion

1 Introduction

Electrostrictive materials receive growing attention due to their application as sensors, actuators [1] or transducers [2]. They belong to the class of electroelastic materials exhibiting a quadratic dependence of stress fields upon electric fields.

E. Danicki (✉) • Y. Tasinkevych
Department of Physical Acoustics, Institute of Fundamental Technological Research of the Polish Academy of Sciences, 5B Pawińskiego str., Warsaw 02-106, Poland
e-mail: edanicki@ippt.gov.pl

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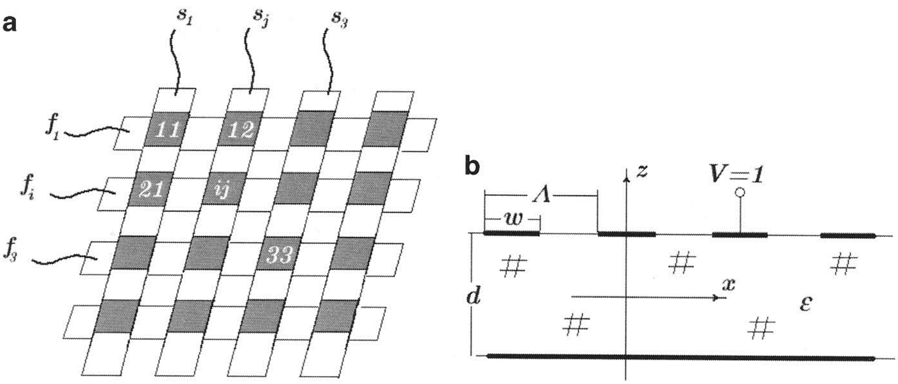

Fig. 1 (**a**) Periodic metal strips (electrodes) arranged perpendicularly on both sides of electrostrictive layer and connected to external voltage sources. (**b**) The voltage $V = 1$ is applied to the upper lth strip residing on a d-thick dielectric layer; other strips are grounded

Typically, uniform electric field is applied to entire device, being a plate or membrane made of electroactive polymers and placed between two compliant electrodes, causing its uniform deformation [3]. Here, we discuss an arbitrary nonuniform electric field resulting in intentional nonuniform stress in the plate and its nonuniform vibrations. The considered system consists of an electrostrictive dielectric layer with two systems of parallel conducting electrodes arranged on both surfaces oriented at the right angle to each other. Such a system is shown to be suitable for addressable driving of two-dimensional matrix transducer comprising the above crossed periodic metal strips representing the matrix rows and columns. Perspective application of such a device may be in three-dimensional ultrasound imaging systems. The typical two-dimensional acoustic transducers used in applications, however, require a large number of channels to be connected to individual elements, which causes serious difficulties in manufacturing of the driving circuits and corresponding wiring, especially at higher operating frequencies. The system considered here substantially reduces the number of signal channels from N^2 to 2 N for $N \times N$ element two-dimensional transducer array. A similar structure was earlier considered in [4] where the authors proposed a two-dimensional array of edge-connected crossed electrodes. In [5] a crossed-electrode transducer array was studied in the signal processing framework. However no profound theoretical analysis of the considered crossed-electrode array has been carried out so far. In this paper the theoretical model of considered structure is developed using the method earlier developed in electrostatics. To evaluated the stress in the layer excited by potentials applied to electrodes the formulation of nontrivial electrostatic problem is required. Its solution is based on application of the BIS expansion method which was earlier successfully used for solving electrostatic problems in the theory of surface acoustic waves interdigital transducers [6].

Consider a structure containing an electrostrictive dielectric layer with dielectric permittivity ε and the systems of parallel conducting strips placed on the opposite surfaces and oriented at the right angle to each other, as shown in Fig. 1a.

expanded into a similar series of spatial harmonics with corresponding amplitudes D_{nm}. The boundary conditions on the upper (superscript u) and bottom (superscript b) surfaces of the dielectric layer imposed on the field components are [10]:

$$\begin{aligned} E_x^u = 0, \quad E_y^b = 0, \quad &\text{on strips,} \\ D^u = 0, \quad D^b = 0, \quad &\text{between strips.} \end{aligned} \tag{4}$$

Applying the BIS expansion as in [7] the surface fields satisfying conditions (4) are:

$$\begin{aligned} D^u &= j\varepsilon \sum_{n',n,m} \alpha_{n'}^m P_{n-n'}(\Delta) \mathrm{e}^{-j(r_n x + s_m y)}, \\ E_x^u &= \sum_{n',n,m} \alpha_{n'}^m S_{n-n'} P_{n-n'}(\Delta) \mathrm{e}^{-j(r_n x + s_m y)}, \\ D^b &= -j\varepsilon \sum_{m',n,m} \beta_{m'}^n P_{m-m'}(\Delta) \mathrm{e}^{-j(r_n x + s_m y)}, \\ E_y^b &= \sum_{m',n,m} \beta_{m'}^n S_{m-m'} P_{m-m'}(\Delta) \mathrm{e}^{-j(r_n x + s_m y)}, \end{aligned} \tag{5}$$

where $\Delta = \cos(Kw/2)$, P_k are the Legendre polynomials. The unknown coefficients $\alpha_{n'}^m, \beta_{m'}^n$ are evaluated from the relation between spatial spectra of the tangential electric field $E^{u,b}$ and normal electric induction $D^{u,b}$ on the dielectric surfaces, governing the field inside the layer:

$$\begin{bmatrix} E^u \\ E^b \end{bmatrix} = \frac{S_k}{j\varepsilon} \begin{bmatrix} \coth|k|d & -1/\sinh|k|d \\ 1/\sinh|k|d & -\coth|k|d \end{bmatrix} \begin{bmatrix} D^u \\ D^b \end{bmatrix}. \tag{6}$$

Apparently, the field expansion Eq. 5 must obey Eq. 6 for any Bloch component having wave-numbers $k = k_{nm}$ included in the expansion. It is worth noting, that the higher Bloch orders harmonics vanish fast inside the layer and are negligible on the opposite surface of the layer due to fast vanishing term $1/(\sinh k_{nm}d)$. Thus, for large k_{nm} the corresponding spatial harmonics are well-localized at a given dielectric surface. This significantly simplifies the analysis due to the equations separation for large k_{nm}.

Substitution of the Bloch components from Eq. 5 into Eq. 6 for $m \in [-M, 0]$, $n \in [-N, N-1]$, with $n' \in [-N, N-1]$ and $m' \in [-M, 0]$ (note, the symmetry properties of the coefficients $\beta_{1-m'}^n = \beta_{m'}^n$ and $\alpha_{n'}^{-m} = \alpha_{n'}^m$ are used here) yields the following conditions for unknown coefficients $\alpha_{n'}^m, \beta_{m'}^n$:

$$\begin{aligned} \alpha_{n'}^m \left[S_{n-n'} \tanh k_{nm}d - \frac{r_n}{k_{nm}} \right] P_{n-n'} - \beta_{m'}^n \frac{r_n}{k_{nm}} \frac{P_{m-m'} - P_{-m-m'}}{\cosh k_{nm}d} &= 0, \\ -\alpha_{n'}^m \frac{s_m}{k_{nm}} \frac{P_{n-n'}}{\cosh k_{nm}d} + \beta_{m'}^n \left[S_{m-m'} \tanh k_{nm}d - \frac{s_m}{k_{nm}} \right] P_{n-n'} &= 0. \end{aligned} \tag{7}$$

In Eq. 7 $S_\nu = \text{sign}(\nu + 0)$ and $P_\nu = P_\nu(\Delta)$ is applied to shorten notation. The truncation numbers involved in the above system of equations are determined by the BIS-expansion conditions [7]. Generally, both N, M should be infinite, but practically it is sufficient to apply N, M not very large finite integers. Let N', M' be such that $\tanh N'Kd \approx \tanh M'Kd \approx 1$then $N > N'$ and $M > M'$ should be chosen such that $r_N/k_{NM'} \approx 1$and $s_M/k_{N'M} \approx 1$ respectively.

In the above system the last equation for $m = 0$ should be replaced with the following:

$$\frac{-P_{n-n'}}{\cosh kd}\alpha_{n'}^{0} + 2\left[(-1)^{m'}\frac{k}{K}\tanh kd\frac{d}{d\xi}P_{-m'+\xi}(-\Delta)|_{\xi=0} - P_{-m'}\right]\beta_{m'}^{n} = 0, \quad \text{(7a)}$$

where $k = k_{n0}$. Integrating corresponding tangential components of the electric field one obtains the potential distribution at the plane of strips on the upper and bottom sides. In the considered boundary-value problem the potentials of strips are given as additional constrains to determine unknown expansion coefficients uniquely. For the case, shown in Fig. 1b, the unitary voltage is applied to l^{th} upper strip and all the bottom strips assumed to be grounded ($s_m = mK$) this condition results in:

$$(-1)^{n'}\alpha_{n'}^{m}P_{-n'-r/K}(-\Delta) = \delta_{m0}\frac{K}{\pi}e^{jrl\Lambda}\sin\frac{\pi r}{K}, \quad \text{(8)}$$

where δ_{ij} is Kronecker delta. Solving Eqs. 7, 7′, and 8 for $\alpha_{n'}^{m}, \beta_{m'}^{n}$ the planar electric field can be determined on both surfaces of dielectric layer from Eq. 5.

3 Electrostrictive Stress in the Layer

According to Eq. 2, the electrostrictive stress in the layer $\sigma^{(ij)}$ or, more exactly, its z-component, considered here, is proportional to the product of $E_z(x,y)$ component of electric field, resulting from the applied potential to the upper i^{th} electrode and $E_z(x,y)$ excited by the bottom j^{th} electrode. In the considered case of the same strip periodicity and width on both upper and bottom sides of the layer, the latter equals $E_z(y,x)$. It is known [11, 12] that the electric field is singular at the strip edges (this is the cost of idealization of real, finite-thickness electrodes by strips of infinitesimal thickness). In order to avoid the corresponding difficulty, we evaluate the E_z components of electric field at the layer middle plane $z = 0$. It can be reconstructed from the surface normal induction on both surfaces of the layer. Applying the spectral variable $\rho = r + nK$ spanning entire domain and following the same considerations as in [7] one obtains:

$$E_z = 2\int_{-\infty}^{\infty} e^{-j\rho x}\sum_{m=-\infty}^{\infty}\frac{\alpha_{n'}^{m}P_{n-n'}(\Delta) + \beta_{m'}^{n}P_{m-m'}(\Delta)}{K\sinh(k_{nm}d/2)}e^{-jmKy}. \quad \text{(9)}$$

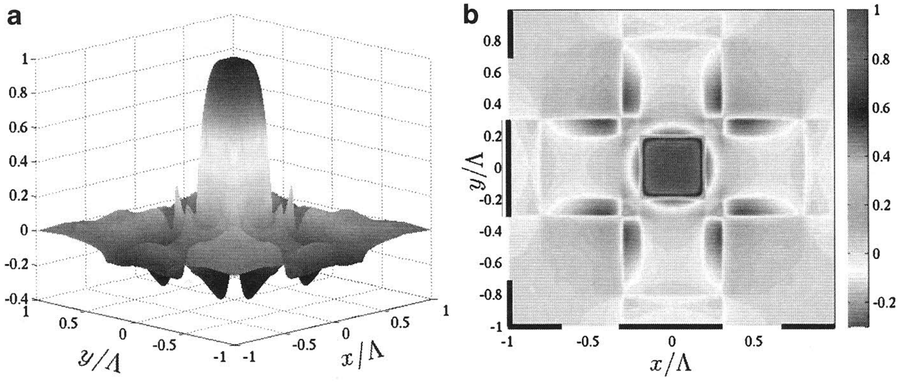

Fig. 2 The electrostrictive stress generated in the $2\Lambda \times 2\Lambda$ domain of the dielectric layer at the plane $z = 0$ for $d/\Lambda = 0.15$, $w/\Lambda = 0.6$

Fast growing term $\sinh(k_{nm}d/2)$makes the above equation suitable for numerical evaluation.

In Fig. 2 the numerical example of the σ_z component of the electrostrictive stress in the layer middle plane $z = 0$ is shown in relative scale.

As clearly shown in Fig. 2, the stress distribution at the middle plane of the dielectric layer significantly departs from uniform and spans somewhat outside the cell covered by the supplied strips.

4 Conclusions

The system of row and column electrodes is proposed for addressable driving of electrostrictive transducer matrix. Detailed analysis of electrostatic field is presented. The same strip parameters Λ, w has been assumed on both sides of the electrostrictive dielectric layer to simplify the presentation. The method can be easily generalized for different strip period and width. Numerical examples show the resulting nonuniform electrostrictive stress induced in the area of the excited matrix cell for one upper strip excited by a uniform voltage and all bottom strips grounded.

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References

1. Pimpin, A., Kasagi, N.: Micro electrostrictive actuator with metal compliant electrodes for flow control applications. In: Proceedings of 17 IEEE International Conference on MEMS, Maastricht, pp. 478–481 (2004)

2. Strachan, A., Goddard, W.A.: Large electrostrictive strain at gigahertz frequencies on a polymer nanoactuator: computational device design. Appl. Phys. Lett. **86**, 083103 (2005)
3. Resset, S., Niklaus, M., Dubois, P., Dadras, M., Shea, H.: Mechanical properties of electroactive polymer microactuators with ion implanted electrodes. Proc. SPIE **6524**, 652410 (2007)
4. Schau, H.C.: Edge-connected, crossed-electrode array for two-dimensional projection and beam forming. IEEE Trans. Signal Process. **39**, 289–297 (1991)
5. Fujishima, I., Tamura, Y., Yanagida, H., Tada, J., Takahashi, T.: Edge-connected, crossed-electrode array comprising non-linear transducer. In: Proceedings of IEEE International Ultrasonics Symposium, Rome, pp. 2221–2224 (2009)
6. Tasinkevych, Y.: Electrostatics of planar system of conducting strips. In: Bertrand, C.L. (ed.) Electrostatics: Theory and Applications, Chap. 7. Nova Science Pub Inc., New York, pp. 189–221 (2011). ISBN: 978-1-61668-549-2
7. Danicki, E.: Electrostatics of crossed arrays of strips. IEEE Trans. Ultrason. Ferroelectr. Freq. Control **67**, 1701–1705 (2010)
8. Danicki, E.: Spectral Theory of Interdigital Transducers, Chap. 3. Manuscript available online at http://www.ippt.gov.pl/~edanicki/danickibook.pdf (2006)
9. Danicki, E.J.: A method for analyzing periodic strips with apodization. IEEE Trans. Ultrason. Ferroelectr. Freq. Control **55**(9), 1890–1894 (2008)
10. Bløtekjær, K., Ingebrigtsen, K.A., Skeie, H.: A method for analyzing waves in structures consisting of metal strips on dispersive media. IEEE Trans. Electron. Device **20**, 1133–1138 (1973)
11. Thakur, O.P., Singh, A.K.: Electrostriction and electromechanical coupling in elastic dielectrics at nanometric interfaces. Mater. Sci. **27**, 839–850 (2009)
12. Jiang, Q., Kuang, Z.-B.: Stress analysis in two dimensional electrostrictive material with an elliptic rigid conductor. Eur. J. Mech. A Solids **23**, 945–956 (2004)